

Error Detection

Regardless of the design of the transmission system, there will be errors, resulting in the change of one or more bits in a transmitted frame. In what follows, we assume that data are transmitted as one or more contiguous sequences of bits, called frames. We define these probabilities with respect to errors in transmitted frames:

P_b: Probability that a bit is received in error; also known as the bit error rate (BER)

P₁: Probability that a frame arrives with no bit errors

P₂: Probability that, with an error-detecting algorithm in use, a frame arrives with one or more undetected errors

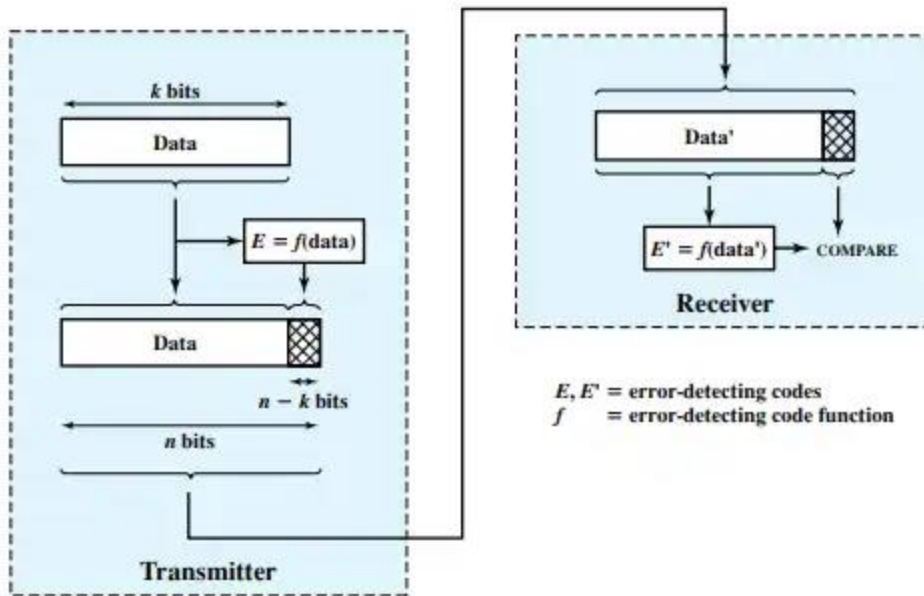
P₃: Probability that, with an error-detecting algorithm in use, a frame arrives with one or more detected bit errors but no undetected bit errors

First consider the case in which no means are taken to detect errors. Then the probability of detected errors is zero. To express the remaining probabilities, assume the probability that any bit is in error is constant and independent for each bit. Then we have

$$P_1 = (1 - P_b)^F$$
$$P_2 = 1 - P_1$$

where F is the number of bits per frame. In words, the probability that a frame arrives with no bit errors decreases when the probability of a single bit error increases, as you would expect. Also, the probability that a frame arrives with no bit errors decreases with increasing frame length; the longer the frame, the more bits it has and the higher the probability that one of these is in error.

This is the kind of result that motivates the use of error-detecting techniques. For a given frame of bits, additional bits that constitute an error-detecting code are added by the transmitter. This code is calculated as a function of the other transmitted bits. Typically, for a data block of k bits, the error-detecting algorithm yields an error-detecting code of bits, where The error-detecting code, also referred to as the check bits, is appended to the data block to produce a frame of n bits, which is then



transmitted. The receiver separates the incoming frame into the k bits of data and bits of the error-detecting code. The receiver performs the same error-detecting calculation on the data bits and compares this value with the value of the incoming error-detecting code. A detected error occurs if and only if there is a mismatch. Thus is the probability that a frame contains errors and that the error-detecting scheme will detect that fact. is known as the residual error rate and is the probability that an error will be undetected despite the use of an error-detecting scheme.

Parity Check

The simplest error-detecting scheme is to append a parity bit to the end of a block of data. A typical example is character transmission, in which a parity bit is attached to each 7-bit IRA character. The value of this bit is selected so that the character has an even number of 1s (even parity) or an odd number of 1s (odd parity).

Note, however, that if two (or any even number) of bits are inverted due to error, an undetected error occurs. Typically, even parity is used for synchronous transmission and odd parity for asynchronous transmission. The use of the parity bit is not foolproof, as noise impulses are often long enough to destroy more than one bit, particularly at high data rates.

Cyclic Redundancy Check (CRC)

One of the most common, and one of the most powerful, error-detecting codes is the cyclic redundancy check (CRC), which can be described as follows. Given a k -bit block of bits, or message, the transmitter generates an sequence, known as a frame check sequence (FCS), such that the resulting frame, consisting of n bits, is exactly divisible by some predetermined number. The receiver then divides the incoming frame by that number and, if there is no remainder, assumes there was no error.³ To clarify this, we present the procedure in three equivalent ways: modulo 2 arithmetic, polynomials, and digital logic.

